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Trim effects on motion stability of submersible vehicles

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TRIM EFFECTS ON MOTION STABILITY OF SUBMERSIBLE VEHICLES

by

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NOMENCLATURE

Symbol:	Definition:
A	Closed loop dynamics matrix for the linearized system
α	Control surface coordination gain
b(x)	Local beam of the hull
B	Vehicle buoyancy
B	Control matrix in state space
C_D	Quadratic drag coefficient
c_1, c_2	Coupled heave and pitch terms
d_q, d_w	Cross flow drag terms
δ_b	Bow plane deflection
δ_s, δ	Stern plane deflection
I_y	Vehicle mass moment of inertia
I_H, I_p	Cross flow drag terms
k_1, k_2, k_3, k_4	Controller gains in θ,w,q, and z
K_0	Feedforward gain
m	Vehicle mass
M	Pitch moment
M_a	Partial derivative of M w.r.t. a
θ	Pitch angle
q	Pitch rate
T_c	Time constant
U	Forward speed
U_o	Nominal speed
w	Heave velocity
W	Vehicle weight
x	State vector
x_B, z_B	Body fixed coordinates of vehicle center of buoyancy
x_G, z_G	Body fixed coordinates of vehicle center of gravity
z	Deviation off the nominal depth
z_{GB}	Vehicle metacentric height
Z	Heave force
Z_a	Partial derivative of Z w.r.t. a

I. INTRODUCTION

The fundamental goal of submarine control is to reach and maintain ordered depth. Experimental designs involve expensive model testing such as Darpa Suboff Model (DTRC Model 5470) [Ref. 6]. Much research has been done in depth control of submarines [Ref. 3,5]. Our goal is to develop an analytic method to determine the stability properties of a design.

The stability of a design will have a significant impact on its responsiveness. A vehicle with a large margin to instability will not be very responsive. The problem becomes one of determining how close to the margins we can get without a total loss of stability. By expanding the scope of our research to include nonlinear terms we are able to define the limits of stability and therefore margins.

Previous studies analyzed stability properties of the system, specially static bifurcations [Ref. 2] and bifurcations to periodic solutions [Ref. 1]. The latter study which is used as a basis for this work, was restricted to level, zero trim flight paths.

The purpose of this thesis is to develop a program for finding the limits of stability for an out of trim submarine at moderate and high speeds. These limits are mapped using a Hopf bifurcation analysis program included in the Appendix.

II. EQUATIONS OF MOTION

The motion of the submersible in the vertical plane can be modeled by four coupled nonlinear differential equations for pitch rate (q), heave velocity (w), pitch angle (θ) and heave (z). With a body fixed coordinate frame at the vehicle's geometric center , we can express Newton's equations of motion as

$$m(\dot{w} - Uq - z_G q^2 - x_G \dot{q}) = Z_{\dot{q}}\dot{q} + Z_{\dot{w}}\dot{w} + Z_{\delta_b}\delta_b + Z_{\delta_s}\delta_s + Z_w w + Z_q q - \frac{1}{2}\rho \int C_D b(x) \frac{(w - xq)^3}{|w - xq|} dx \tag{2.1}$$

$$I_y - m x_G(\dot{w} - Uq) - z_G w q m = -x_{GB} B \cos\theta - z_{GB} B \sin\theta + M_{\delta_s}\delta_s$$
$$M_{\delta_b}\delta_b + M_{\dot{q}}\dot{q} + M_{\dot{w}}\dot{w} + M_q q + M_w w + \frac{1}{2}\rho \int C_D b(x) \frac{(w - xq)^3}{|w - xq|} x dx \tag{2.2}$$

$$\dot{\theta} = q \tag{2.3}$$

$$\dot{z} = -U \sin\theta + w \cos\theta \tag{2.4}$$

Equations 2.1 through 2.2 can be written in a more compact form as,

$$\dot{w} = a_{11}Uw + a_{12}Uq + a_{13}z_{GB}\sin\theta + a_{13}x_{GB}\cos\theta + b_1U^2\delta_s \\ b_2U^2\delta_b + d_w(w,q) + c_1(w,q) \tag{2.5}$$

$$\dot{q} = a_{21}Uw + a_{22}Uq + a_{23}z_{GB}\sin\theta + a_{23}x_{GB}\cos\theta + b_1U^2\delta_s \\ b_2U^2\delta_b + d_q(w,q) + c_2(w,q) \tag{2.6}$$

where,

$$D_v = (m - Z_{\dot{w}})(I_y - M_{\dot{q}}) - (mx_G + Z_{\dot{q}})(mx_G + M_{\dot{w}})$$

$$a_{11}D_v = (Z_w - 2C_DE_0U\tan\theta_0)(I_y - M_{\dot{q}}) + (mx_G + Z_{\dot{q}})(M_w + 2C_DE_1U\tan\theta_0)$$

$$a_{12}D_v = (m + Z_q + 2C_DE_1U\tan\theta_0)(I_y - M_{\dot{q}}) + \\ (mx_G + Z_{\dot{q}})(M_q - mx_G - mz_GU\tan\theta_0 - 2C_DE_2U\tan\theta_0)$$

$$a_{21}D_v = (M_w + 2C_DE_1U\tan\theta_0)(m - Z_{\dot{w}}) + (mx_G + M_{\dot{w}})(Z_w - 2C_DE_0U\tan\theta_0)$$

$$a_{22}D_v = (M_q - mx_G - mz_GU\tan\theta_0 - 2C_DE_2U\tan\theta_0) \\ (mx_G + M_{\dot{w}})(m + Z_q + 2C_DE_1U\tan\theta_0) \tag{2.7}$$

$$a_{23}D_v = -(m + Z_{\dot{w}})B$$

$$a_{13}D_v = -(mx_G + Z_{\dot{q}})B$$

$$b_1D_v = (I_y - M_{\dot{q}})Z_\delta - (_G + Z_{\dot{q}})M_\delta$$

$$b_2 D_v = (m - Z_{\dot{w}}) M_\delta + (mx_G + M_{\dot{w}}) Z_\delta$$

$$d_q(w,q) D_v = (m - Z_{\dot{w}}) I_q + (mx_G + M_{\dot{w}}) I_w$$

$$d_w(w,q) D_v = (I_y - M_{\dot{q}}) I_w - (mx_G + Z_{\dot{q}}) I_q$$

$$c_1(w,q) D_v = (I_y - M_{\dot{q}}) m z_G q^2 - (mx_G + Z_{\dot{q}}) m z_G wq$$

$$c_2(w,q) D_v = -(m - Z_{\dot{w}}) m z_G w q - (mx_G + M_{\dot{w}}) m z_G q^2$$

In these equations the submersible is assumed to be neutrally buoyant (W=B), and statically stable ($z_G > z_B$). Here we can assume z_B to be zero, hence $z_{GB} = z_G$.

At steady state the cross flow drag integral terms I_H and I_P have the form,

$$I_H = -C_D w |w| \int b(x)\, dx \qquad I_P = C_D w |w| \int b(x)\, x\, dx \qquad (2.8)$$

From equation (2.3) it is seen that w is equal to $\tan\theta_0$ at steady state. The $\int b(x)dx$ term is computed numerically for the SUBOFF model as E_0, and $\int b(x)xdx$ term as E_1. Therefore, the cross flow drag terms become,

$$I_H = -C_D\, w\, |w|\, E_0 \qquad I_P = C_D\, w\, |w|\, E_1 \qquad (2.9)$$

Because we have two rudders at the bow and the stern, our system of equations is multi-input. To reduce this system into a single input system the linear combination of the control inputs will be modified into the following form,

$$\delta = \delta_s \quad , \quad \delta_b = \alpha\delta_s \tag{2.10}$$

where α is the control surface coordination gain. The value of α ranges from -1 to 1. The selection of the value of α will allow the planes to operate as desired for the particular maneuvering condition, i.e., $\alpha = 0$ for no bow plane control, $\alpha = -1$ for bow plane and stern plane opposed to each other, yielding the maximum pitch moment, and $\alpha = 1$ for bow and stern plane control in the same direction, yielding the maximum heave force.

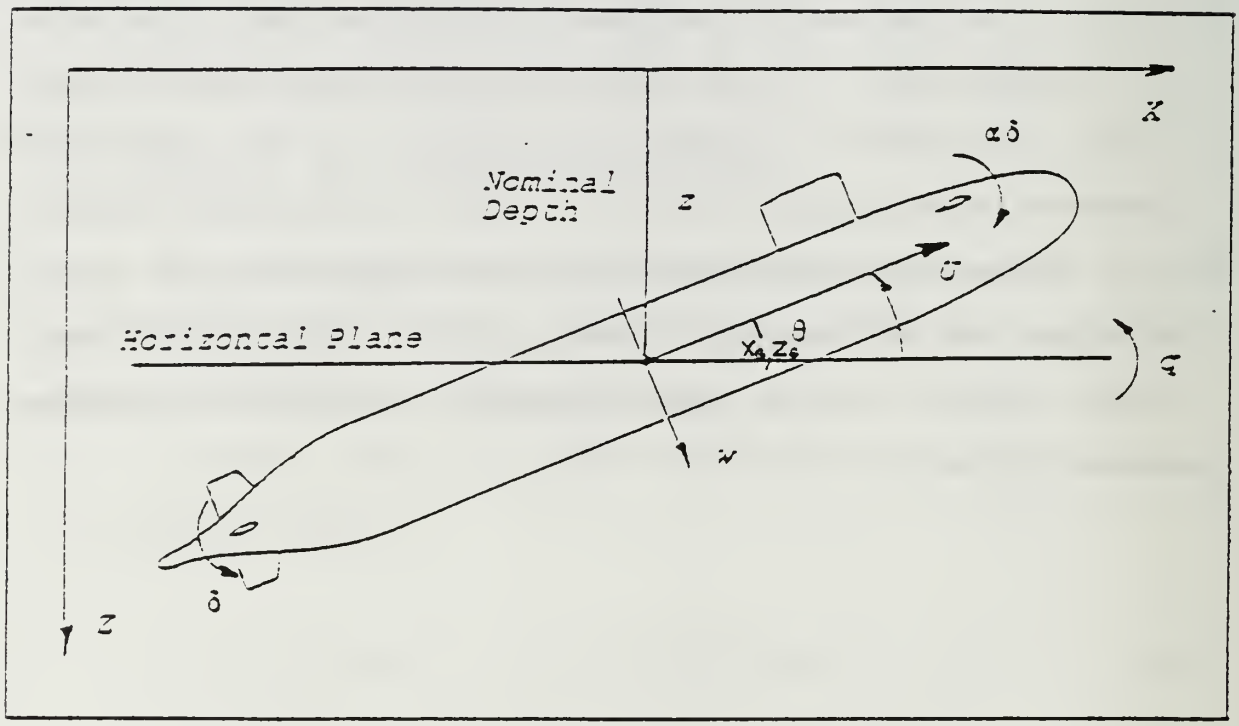

FIGURE 2.1 Geometric representations of the basic definitions.

III. CONTROL LAW

A. INTRODUCTION

The control design problem can be expressed in state space as follows,

$$\dot{x} = Ax + B\delta \tag{3.1}$$

where the state vector is

$$x = \begin{bmatrix} \theta \\ w \\ q \\ z \end{bmatrix} \tag{3.2}$$

Equation 3.1 in our case is,

$$\begin{bmatrix} \dot{\theta} \\ \dot{w} \\ \dot{q} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ a_{13}z_{GB} - b_1u^2k_1 & a_{11}u - b_1u^2k_2 & a_{12}u - b_1u^2k_3 & -b_1u^2k_4 \\ a_{23}z_{GB} - b_2u^2k_1 & a_{21}u - b_2u^2k_2 & a_{22}u - b_2u^2k_3 & -b_2u^2k_4 \\ -u & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ w \\ q \\ z \end{bmatrix} \tag{3.3}$$

Our aim is to find a controller which will assure us a stable closed loop system. The only control input is the dive plane angle, δ.

B. FEEDBACK CONTROL

1. Pole Placement

The full state feedback controller is a linear function of the states and has the form,

$$\delta = - K x \tag{3.4}$$

where K is the vector of feedback gains which are to be determined in order to give the desired closed loop system dynamics. Substituting equation 2.12 into 2.10 yields,

$$\dot{x} = (A - BK) x \tag{3.5}$$

The feedback gains K must be chosen such that A - BK has the desired eigenvalues. The actual characteristic equation of the closed loop system is given by,

$$det(A - BK - sI) = 0 \tag{3.6}$$

The required values of K are obtained by matching coefficients in the two polynomials of the actual and the desired characteristic equations. Equation 3.5 becomes,

$$\begin{bmatrix} \dot{\theta} \\ \dot{w} \\ \dot{q} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ a_{13}z_{GB} & a_{11}u & a_{12}u & 0 \\ a_{23}z_{GB} & a_{21}u & a_{22}u & 0 \\ -u & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ w \\ q \\ z \end{bmatrix} + \begin{bmatrix} 0 \\ b_1 u^2 \\ b_2 u^2 \\ 0 \end{bmatrix} \delta \tag{3.7}$$

The characteristic equation of the closed loop system is,

$$det\begin{bmatrix} -s & 0 & 1 & 0 \\ a_{13}z_{GB} - b_1u^2k_1 & a_{11}u - b_1u^2k_2 - s & a_{12}u - b_1u^2k_3 & -b_1u^2k_4 \\ a_{23}z_{GB} - b_2u^2k_1 & a_{21}u - b_2u^2k_2 & a_{22}u - b_2u^2k_3 - s & -b_2u^2k_4 \\ -u & 1 & 0 & -s \end{bmatrix} \tag{3.8}$$

which reduces to,

$$s^4 + (A_2k_2 + A_3k_3 - E_1)s^3 + (-B_1k_1 - B_2k_2 - B_3k_3 - B_4k_4 - E_2)s^2 + (-C_1k_1 - C_2k_2 - C_3k_3 - E_3)s + (-D_1k_4 - D_2k_4) = 0 \tag{3.9}$$

where,

$$A_2 = -B_4 = b_1 u^2$$

$$A_3 = -B_1 = b_2 u^2$$

$$B_2 = (a_{22} b_1 - a_{12} b_2) u^3$$

$$B_3 = C_1 = (a_{11} b_2 - a_{21} b_1) u^3$$

$$C_2 = D_1 = (a_{23} b_1 - a_{13} b_2) z_{GB} u^2 \tag{3.10}$$

$$C_4 = (a_{22} b_1 + b_2 - a_{12} b_2) u^3$$

$$D_2 = (a_{11} b_2 - a_{21} b_1) u^4$$

$$E_1 = (a_{11} + a_{22}) u$$

$$E_2 = a_{23}z_{GB} + (a_{12}a_{21} - a_{11} a_{22}) u^2$$

$$E_3 = (a_{13} a_{21} - a_{11} a_{23}) z_{GB} u$$

Now, let's assume that we want to place the closed loop poles at $-p_1$, $-p_2$, $-p_3$, $-p_4$ to have the desired system response. Then the desired characteristic equation is,

$$s^4 + \alpha_1 s^3 + \alpha_2 s^2 + \alpha_3 s + \alpha_4 = 0 \tag{3.11}$$

where,

$$\alpha_1 = p_1 + p_2 + p_3 + p_4$$

$$\alpha_2 = p_1 p_2 + p_1 p_3 + p_1 p_4 + p_2 p_3 + p_2 p_4 + p_3 p_4 \qquad (3.12)$$

$$\alpha_3 = p_1 p_2 p_3 + p_1 p_2 p_4 + p_1 p_3 p_4 + p_2 p_3 p_4$$

$$\alpha_4 = p_1 p_2 p_3 p_4$$

The feedback gains can now be computed by equating the coefficients of equation 3.9 and 3.11,

$$A_2 k_2 + A_3 k_3 = -\alpha_1 - E_1$$

$$B_1 k_2 + B_2 k_2 + B_3 k_3 + B_4 k_4 = \alpha_2 + E_2 \qquad (3.13)$$

$$C_1 k_1 + C_2 k_2 + C_4 k_4 = \alpha_3 + E_3$$

$$(D_1 + D_2) k_4 = \alpha_4$$

We established the method for placing the poles of the system, but we also need to know the desired locations of the poles.

2. Pole Location Selection

In a typical second order system control law design , transient response specifications are given. This results in an allowable region in the s-plane where the desired location of the poles can be obtained. For higher order systems the concept of dominant roots can be employed. In selecting poles the physics of the system must be considered. If the poles are too negative, a small time constant will result, and the system may not be able to react that fast. If we use big gains

K, this also means that the control effort will be large. In practice there are limits based on actuator size and saturation.

Considering the control law design to stabilize the submarine to a level flight path at $\theta = 0$ it is required that the submarine return to the level flight, after some small disturbances in θ or z, within the time it takes for the vehicle to travel three ship lengths. Since our model is 14 feet long and its velocity is 5 feet/sec., the required recovery time is about 10 seconds. This means the time constant is 3 seconds and the closed loop poles should be placed at approximately -0.3.

After placing the poles using equation 3.13 the control law is found to be,

$$\delta = 0.9917\ \theta + 0.8333\ w + 0.6026\ q - 0.0351\ z \qquad (3.14)$$

C. FEEDFORWARD CONTROL

The previous discussion on feedback controller assures closed loop stability, but it acts as a regulator in other words takes all the states to zero values. If we have constant disturbances or we want to track some reference value other than zero we can not do this with state feedback alone.

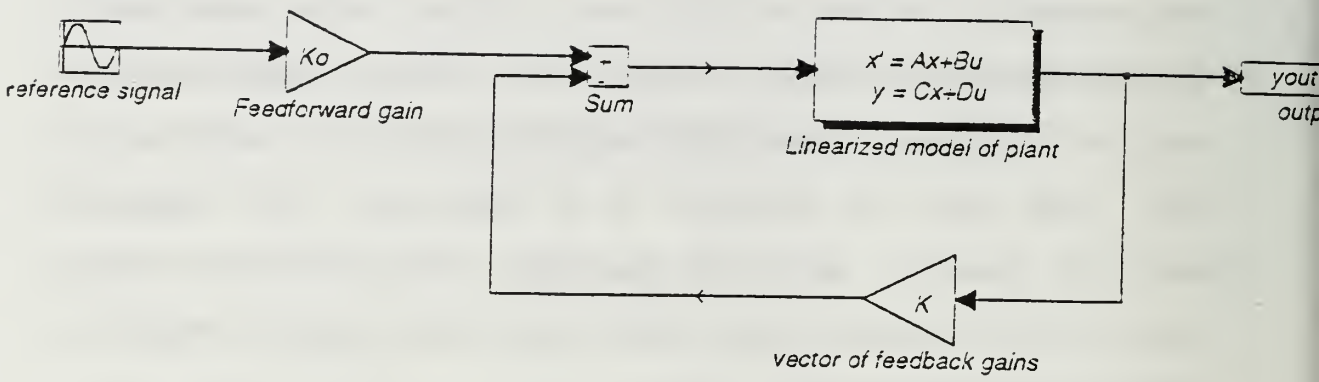

FIGURE 3.1 Feedback and feedforward control application to our linearized model

In the case of non-zero set points or constant disturbances we again need to have the exact same full state feedback to ensure closed loop stability. But we also need to introduce an additional term to our controller in the form

$$u = - K x + k_0 \qquad (3.15)$$

where k_0 is the constant feedforward term. k_0 is given as [Ref. 4],

$$k_0 = H_c^{-1}(0)\ x_0 \tag{3.16}$$

where x_0 is the reference values of the states and $H_c^{-1}(s)$ is the closed loop transfer function,

$$H_c^{-1}(s) = C\ (sI - A + BK)^{-1}\ B \tag{3.17}$$

Another way of getting k_0 is looking at the steady state equations of motion. In steady state all the time derivatives in equations 2.1 through 2.4 go to zero and we have,

$$w = \tan\theta_0 \tag{3.18}$$

$$Z_\delta \delta + Z_w w - I_H = 0 \tag{3.19}$$

$$-(x_G - x_B) B \cos\theta_0 - z_{GB} B \sin\theta_0 + M_\delta \delta + M_w w + I_p = 0 \tag{3.20}$$

If the equation 3.19 is multiplied with M_δ and equation 3.20 with Z_δ and set equal to each other and plug in the equation 3.18, we have an equation depending only on θ.

$$\begin{aligned}(Z_w M_\delta - M_w Z_\delta)\tan\theta_0 + x_G B Z_\delta \cos\theta_0 + z_G B Z_\delta \sin\theta_0 + \\ C_D(E_0 M_\delta - E_1 Z_\delta)\tan\theta_0 |\tan\theta_0|\end{aligned} \tag{3.21}$$

Where E_0 and E_1 are the integral terms computed numerically for the Suboff model. The steady state value of θ_0 is found from equation 3.20 by using a Newton-Raphson method in Bifur1 program in the Appendix. Then we can get δ from

equation 3.19,

$$\delta = \frac{I_H - Z_w w}{Z_\delta} = \frac{-C_D E_0 \tan\theta_0 |\tan\theta_0| - Z_w w}{Z_\delta} \qquad (3.22)$$

After getting δ, we easily find k_0 from equation 3.15,

$$k_0 = \delta + Kx \qquad (3.23)$$

A plot of the steady state angle θ_o versus x_G for different values of z_G is shown in Figure 3.2.

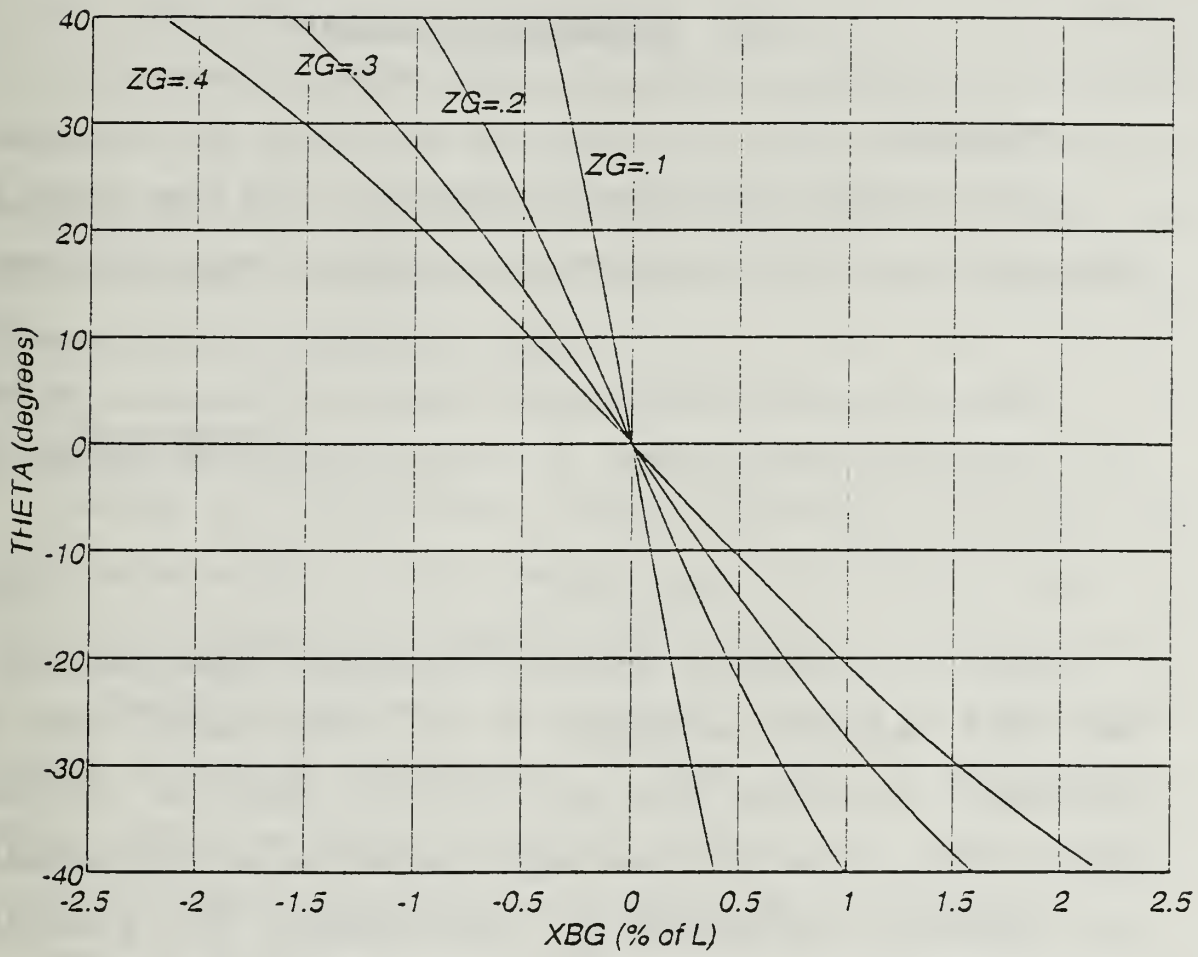

FIGURE 3.2 Steady state pitch angle θ_o as metacentric height varies

IV. BIFURCATION ANALYSIS

A. STABILITY

The nonlinear equations of motion in the dive plane 2.1 through 2.4 can be expressed in a compact form as follows,

$$\dot{x} = f(x) \tag{4.1}$$

Where x is the state variable vector x=[θ,w,q,z]. We know that the equilibrium points, x_0 of the system are defined by,

$$f(x_0) = 0 \tag{4.2}$$

This is a nonlinear system of algebraic equations and it may have multiple solutions in x_0, which means that the nonlinear system may have more than one positions of static equilibrium. If we pick one equilibrium, x_0 we can establish its stability properties by linearization. The linearized system becomes,

$$\dot{x} = Ax \tag{4.3}$$

where A is the Jacobian matrix of f (x) evaluated at x_0,

$$A = \left.\frac{\partial f}{\partial x}\right|_{x_0} \tag{4.4}$$

and the state has been defined to designate small deviations from the equilibrium x_0,

$$x \rightarrow x-x_0 \tag{4.5}$$

In system dynamics as long as all eigenvalues of A have negative real parts, we know that the linear system will be stable. This means that the equilibrium x_0 will be stable for the nonlinear system as well. This is in fact Lyapunov's linearization technique.

B. BIFURCATION

Values of the nonlinear system parameters at which the qualitative nature of the system's motion changes are known as critical or bifurcation values. The phenomena of bifurcation, i.e., quantitative change of parameters leading to qualitative change of system properties, is the topic of bifurcation theory. Euler buckling (Pitchfork bifurcation), limit cycles (Hopf bifurcation) are common examples of bifurcation.

Classical definition of stability states, that the real part of all the eigenvalues of the system must be negative. Therefore, our initial investigations into the stability of the SUBOFF model was to find those eigenvalues whose real parts cross the imaginary axis. We used the bifurcation analysis program, included in the Appendix, to calculate the eigenvalues of the system.

By linearizing the equations of motion, equations 2.1 through 2.4 , the state space equations of the dynamic system can be written in the form,

$$\dot{x} = Ax + Bu \tag{4.5}$$

where,

$$u = - K\ x \tag{4.6}$$

and K is the vector of controller gains, as calculated by pole placement in equations 3.13. The eigenvalues of the system are found by solving,

$$\det|A\text{-}BK\text{-}sI| = 0 \tag{4.7}$$

In the bifurcation program a pseudo-root locus method is employed where the time constant, T_c, is fixed. The constant T_c fixes to placement of the system poles at a given nominal forward speed U_0 and then the model speed, U, is varied incrementally with the system eigenvalues calculated at each speed increment. When the real part of an eigenvalue changes sign between the limits of a speed increment a bisection method is employed to find the speed where the real part of the eigenvalue equals zero.

For each point where the real part of an eigenvalue crosses the imaginary axis the associated T_c and U are plotted on a bifurcation map. This map delineates the regions of

classical stability (all eigenvalues on the left hand plane) from the regions of instability. A family of bifurcation maps were generated by varying nominal speed, U_0, initial stability, z_{GB}, and longitudinal center of gravity/buoyancy separation, x_{GB} of the submersible.

Figure (4.1) shows a typical bifurcation map with its five distinct regions [Ref. 1]. Region I is the area of classical stability. In region II there is one real positive eigenvalue which is indicative of a pitchfork bifurcation. Pitchfork bifurcations of this model were previously examined by Reidel [Ref. 1]. Regions III,IV, and V have at least one pair of complex conjugate eigenvalues with a positive real part. This would indicate that there should be an unstable oscillatory behavior for the model.

C. RESULTS AND DISCUSSION

The classical stability region in bifurcation maps lies between pitchfork and Hopf bifurcation boundaries. The limits or parameters must be defined for the system designer prior to starting the design. By maximizing the region of stability we can give the designer the most leeway in his work. There are

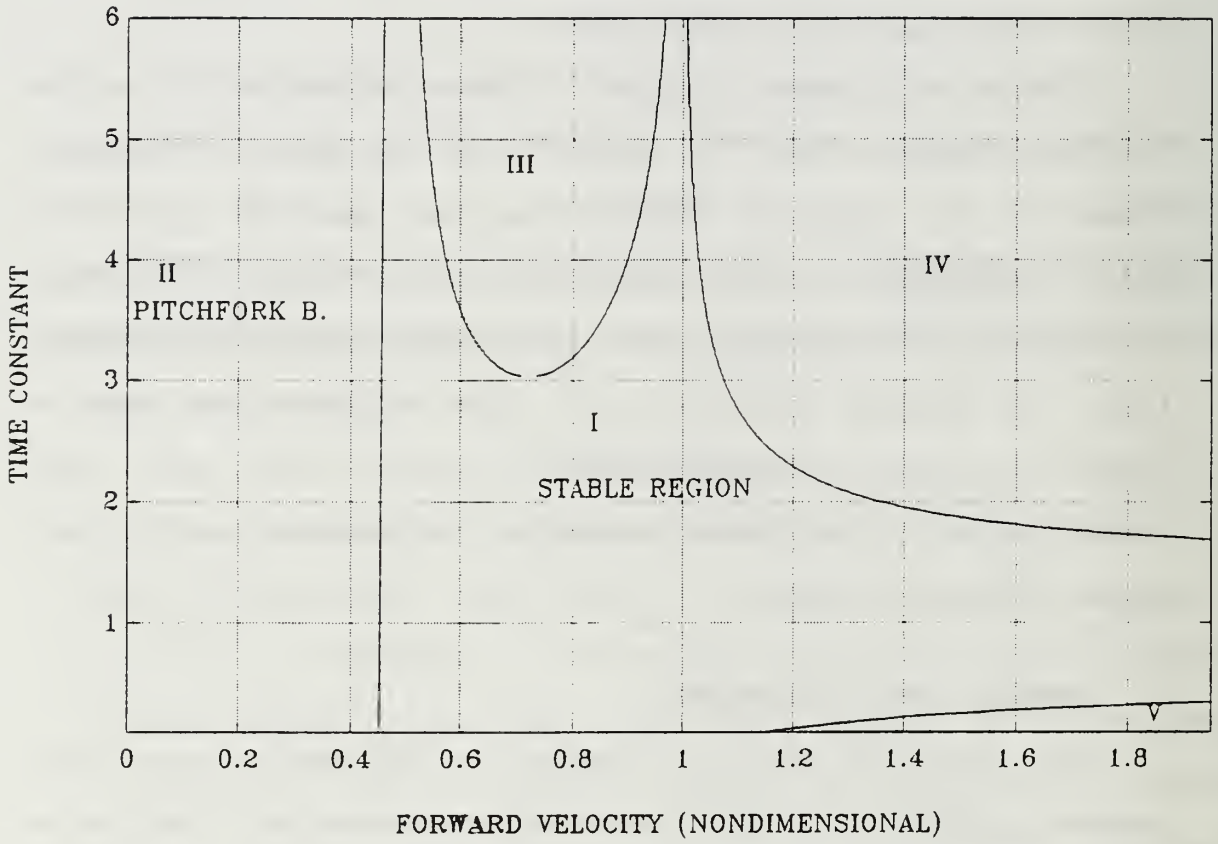

FIGURE 4.1 A typical bifurcation map showing the five distinct regions

two parameters that we used to change the bifurcation maps, the longitudinal separation between center of gravity and center of buoyancy, x_{GB} and the initial stability, z_{GB}.

First we look at the change in x_{GB}. In figures 4.2 through 4.7 we plotted bifurcation curves for different initial stabilities as x_{GB} varies. We can see that as x_{GB} increases the Hopf bifurcation branches H1 an H2 move towards higher speeds and time constants and thus increasing the stability area. The H3 branch however remains constant.

The other important point that we observed is that the system becomes unstable at nominal speed at higher time constants. This is unexpected because we are designing around our nominal speed. A more careful examination in the trimmed case shows that the actual forward velocity becomes, $\sqrt{u^2+w^2}$. Therefore the system may become stable at a value of u other than nominal.

The next parameter we examined was the initial stability, z_{GB}. Figures 4.8 through 4.14 show the effect of varying z_{GB} from .2 to .4 ft for different x_{GB} values. The H3 branch remains constant while the upper speed H2 branch moves down effectively decreasing the area of stability. The low speed curve H1 moves upwards and increases the low speed area of stability.

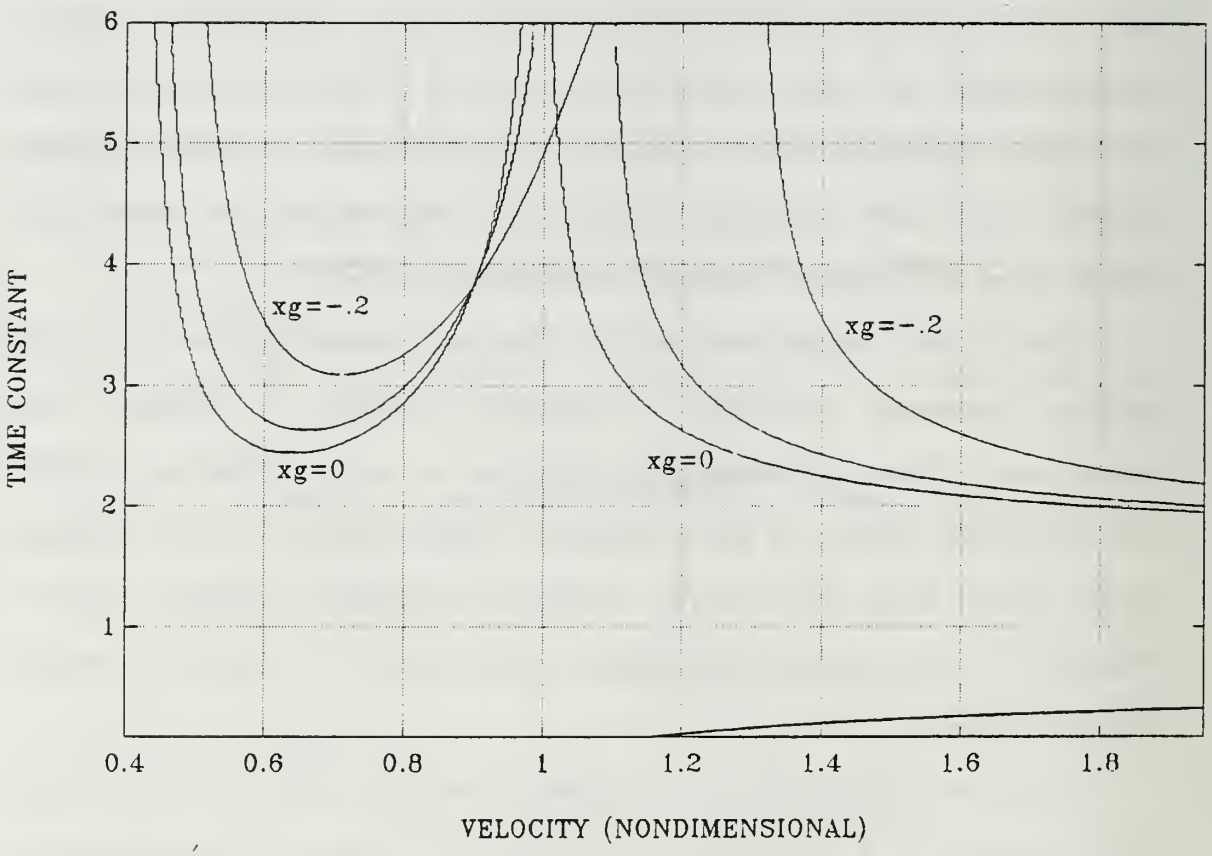

FIGURE 4.2 Bifurcation map as xg changes between xg=0 and xg=-.2, zg=.2.

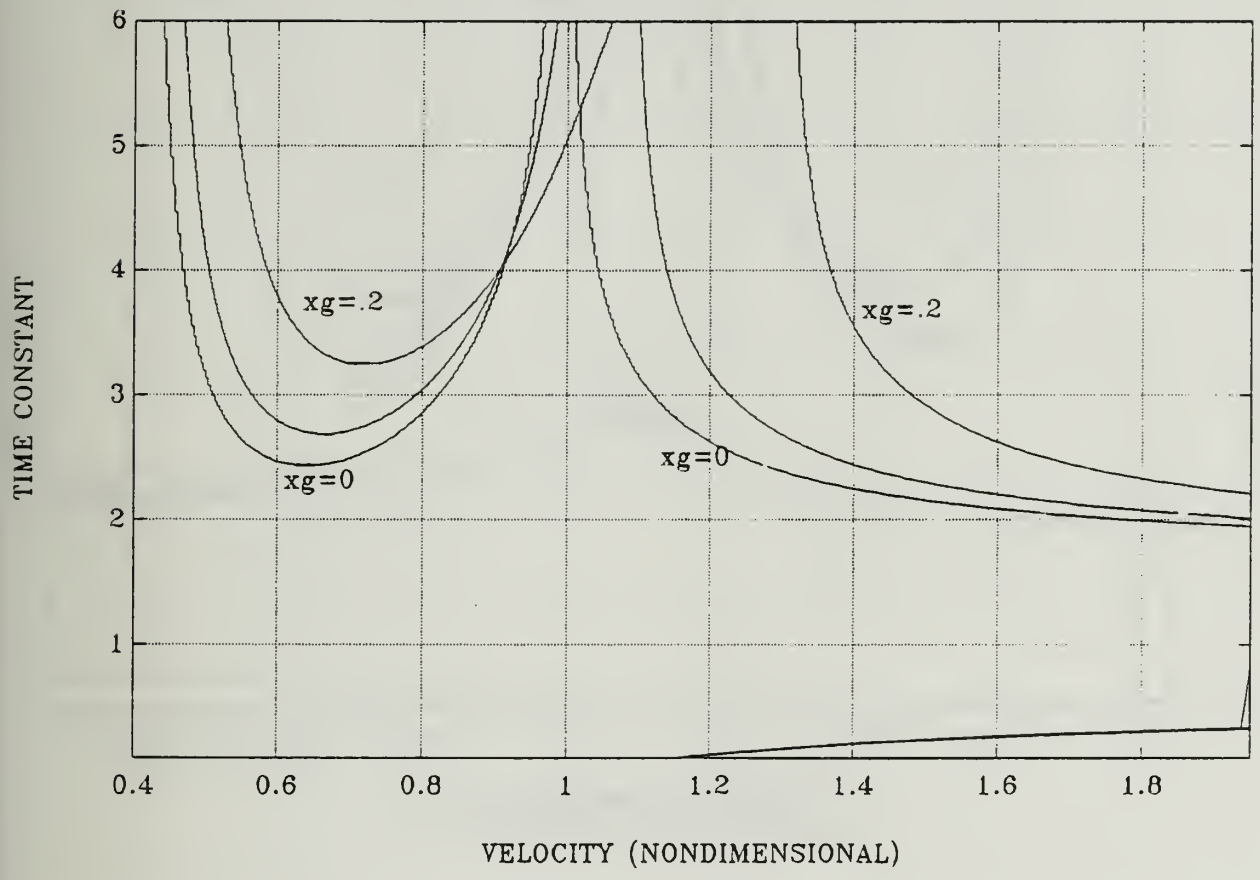

FIGURE 4.3 Bifurcation map as xg changes between xg=0 and xg=.2, zg=.2.

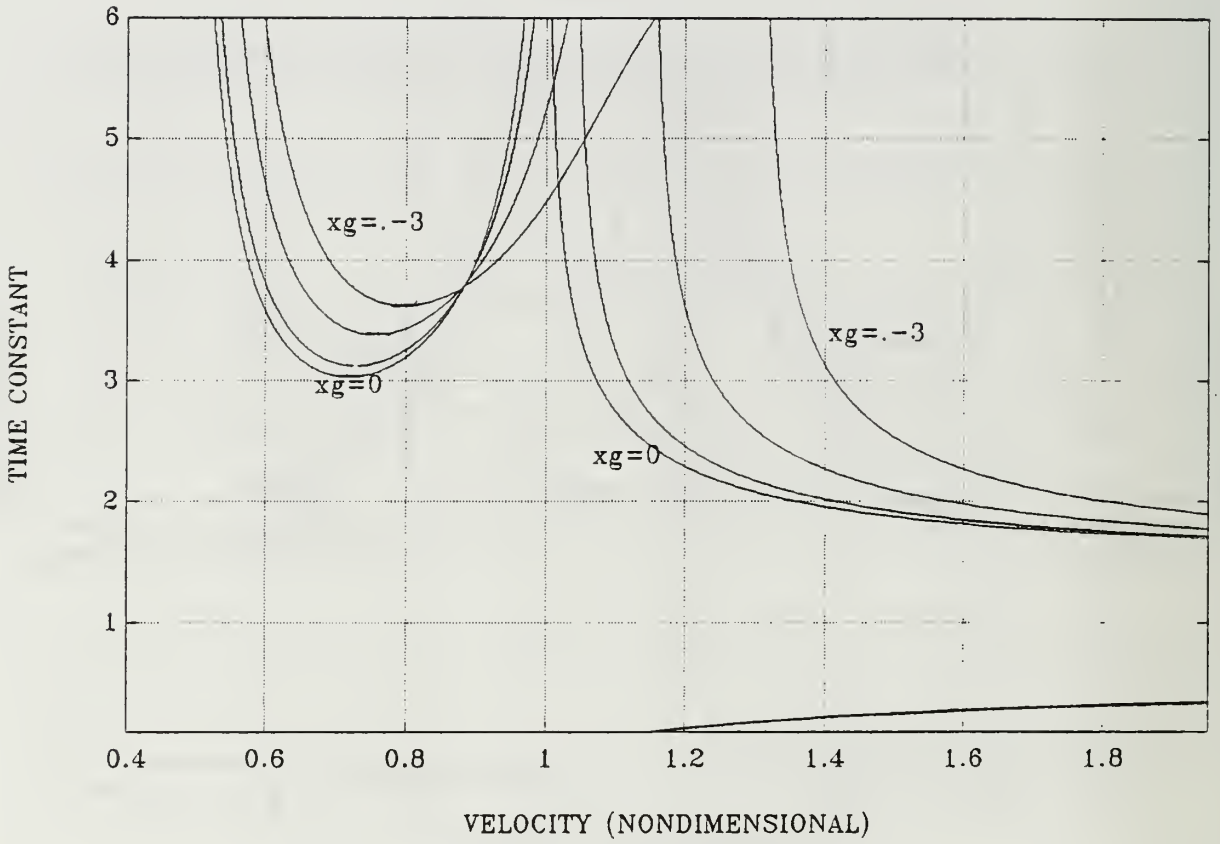

FIGURE 4.4 Bifurcation map as xg changes between xg=0 and xg=-.3, zg=.3.

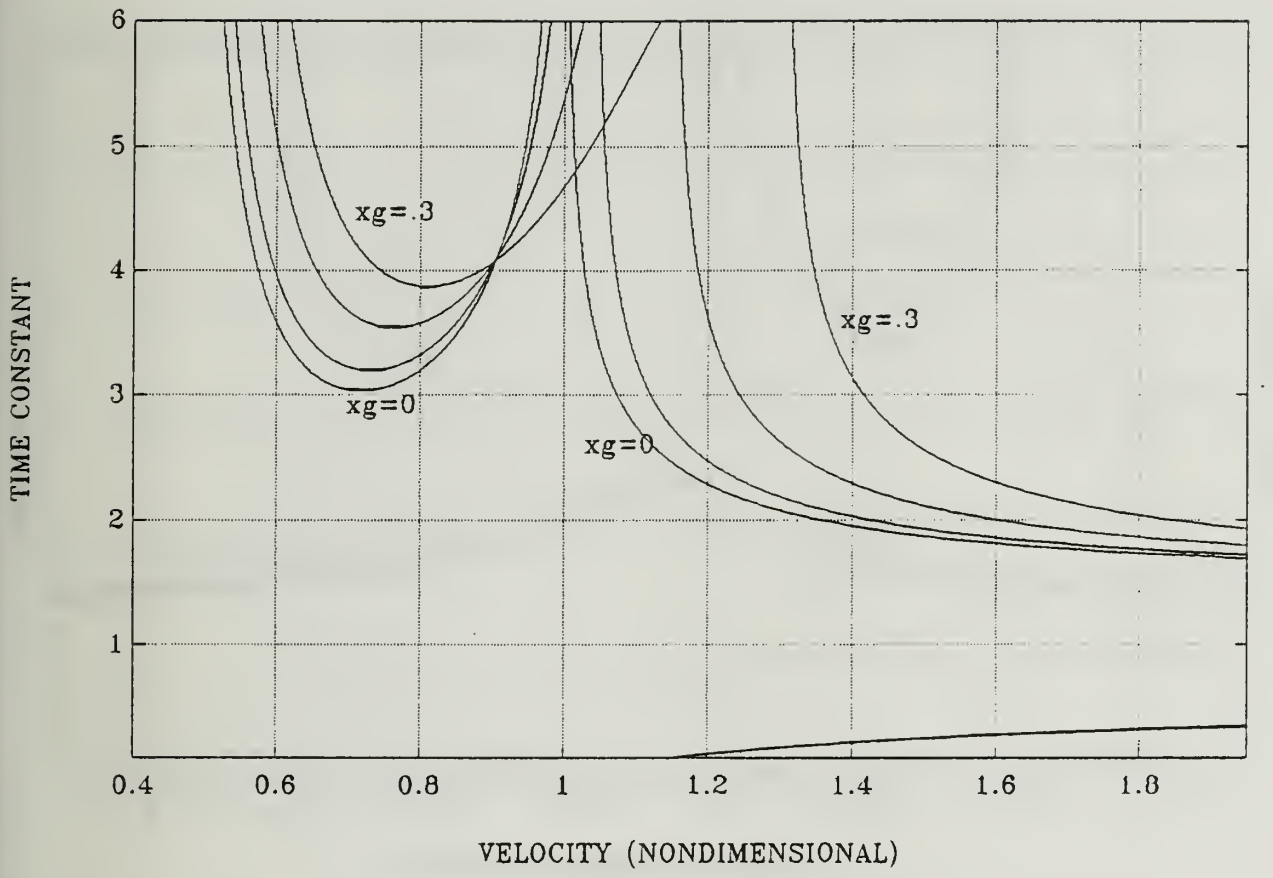

FIGURE 4.5 Bifurcation map as xg changes between xg=0 and xg=.3, zg=.3.

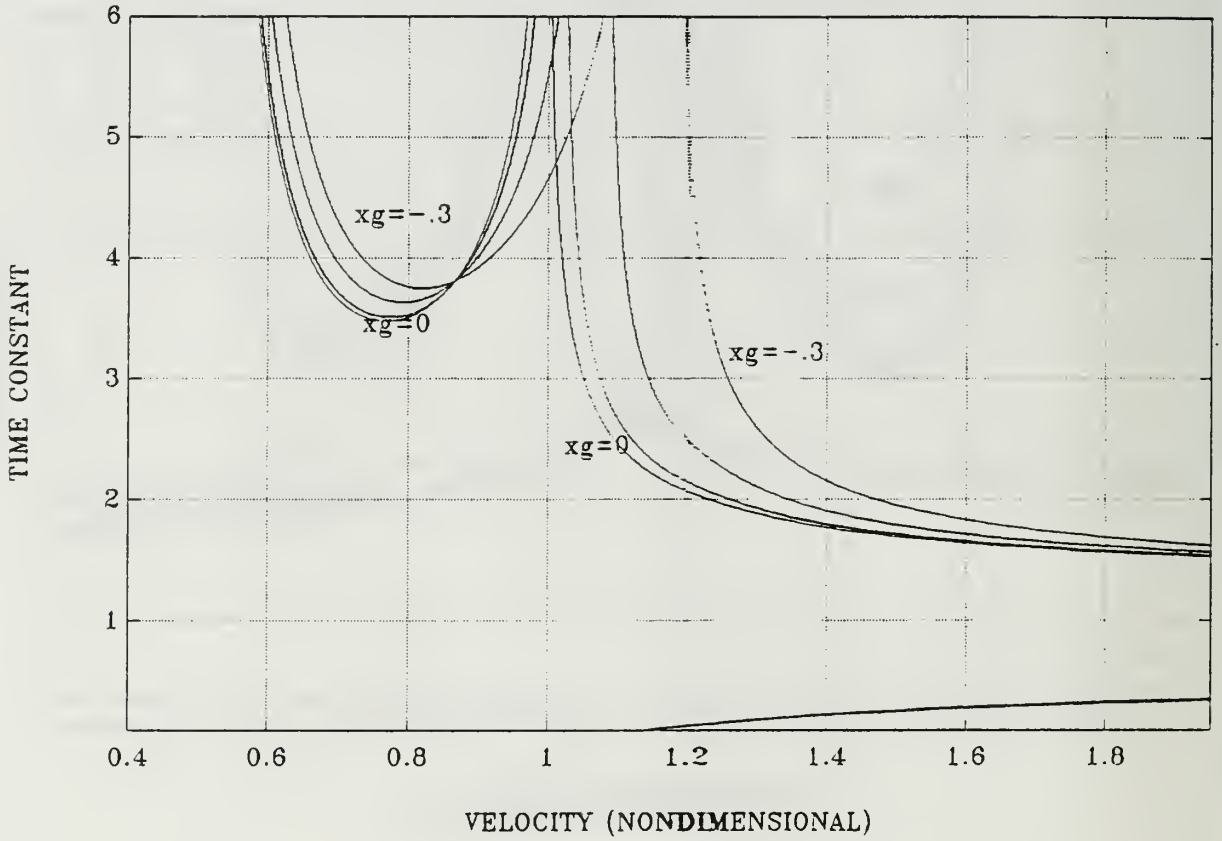

FIGURE 4.6 Bifurcation map as xg changes between xg=0 and xg=-.3, zg=.4.

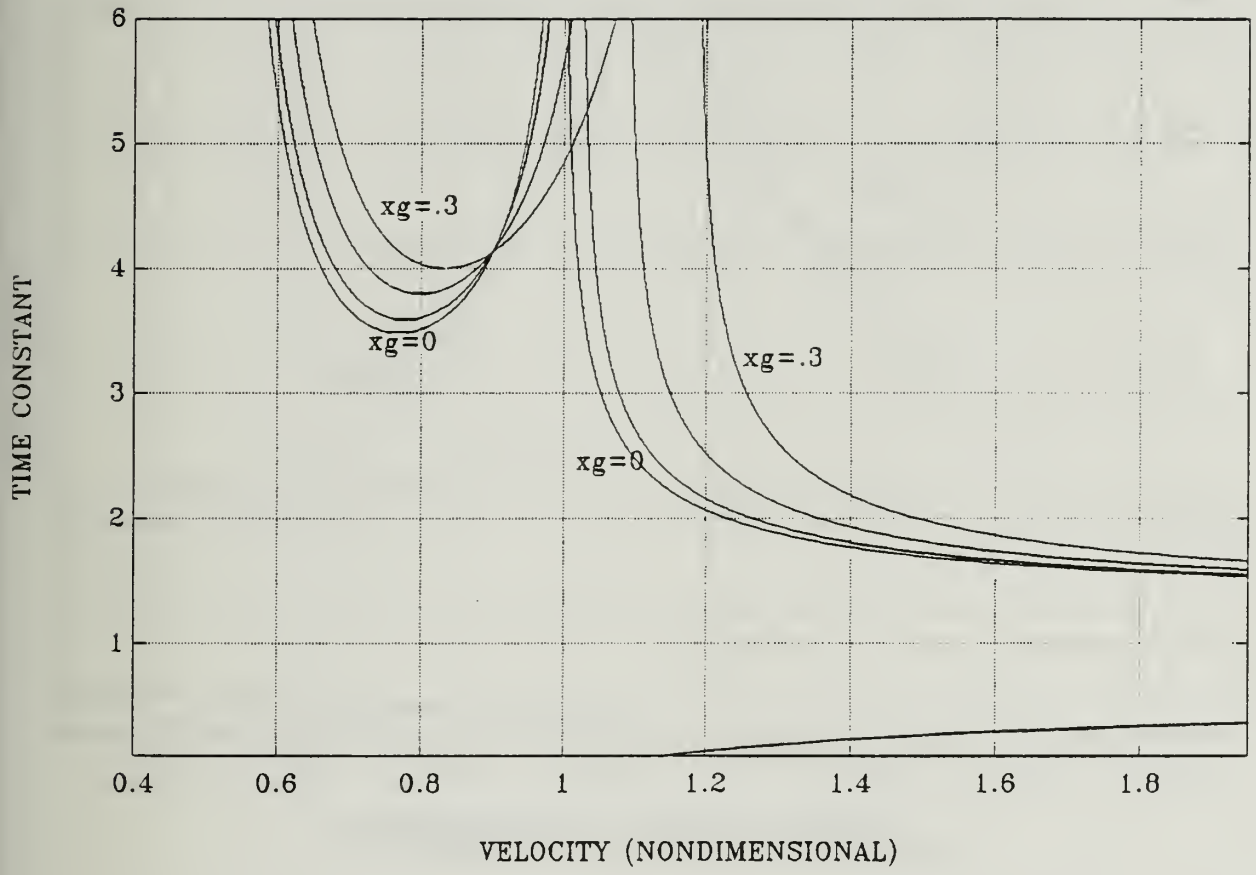

FIGURE 4.7 Bifurcation map as xg changes between xg=0 and xg=.3, zg=.4.

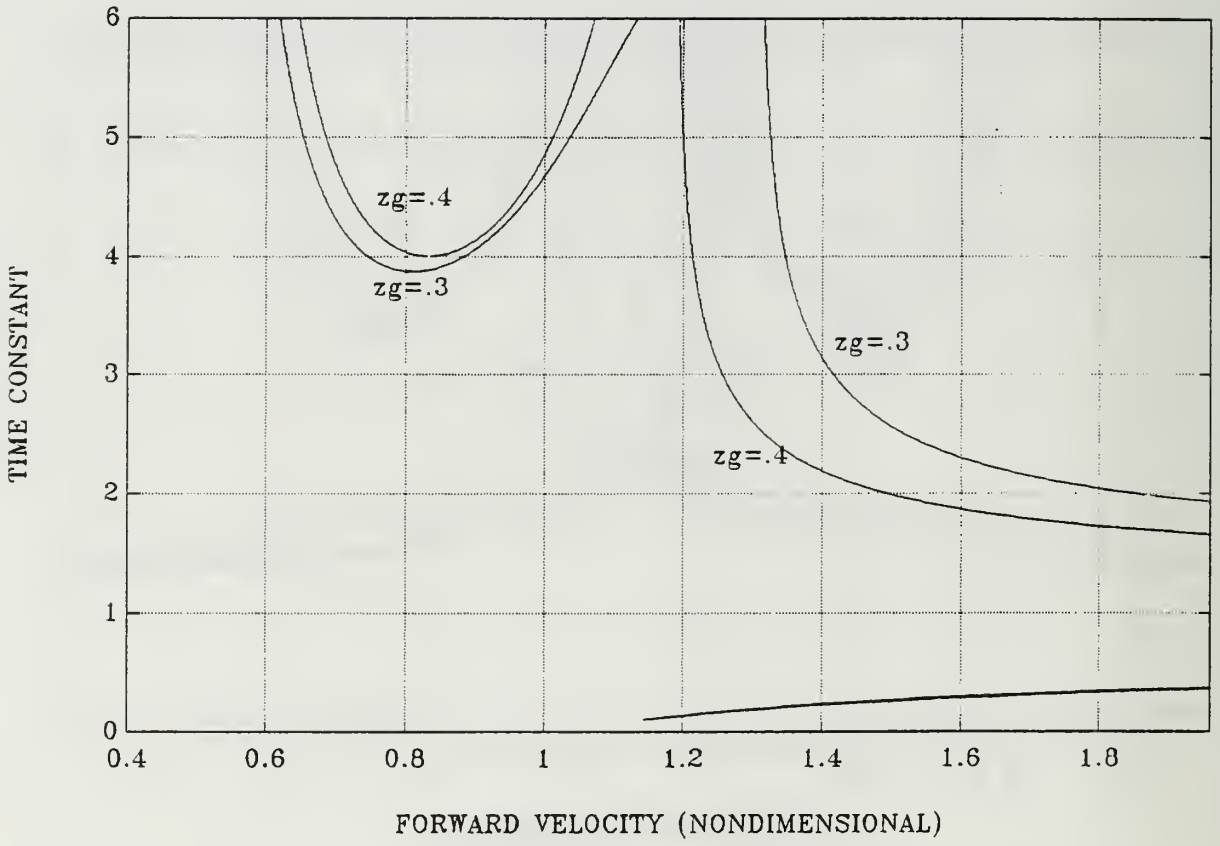

FIGURE 4.8 The effects of changing zg on the bifurcatio maps, xg=.3.

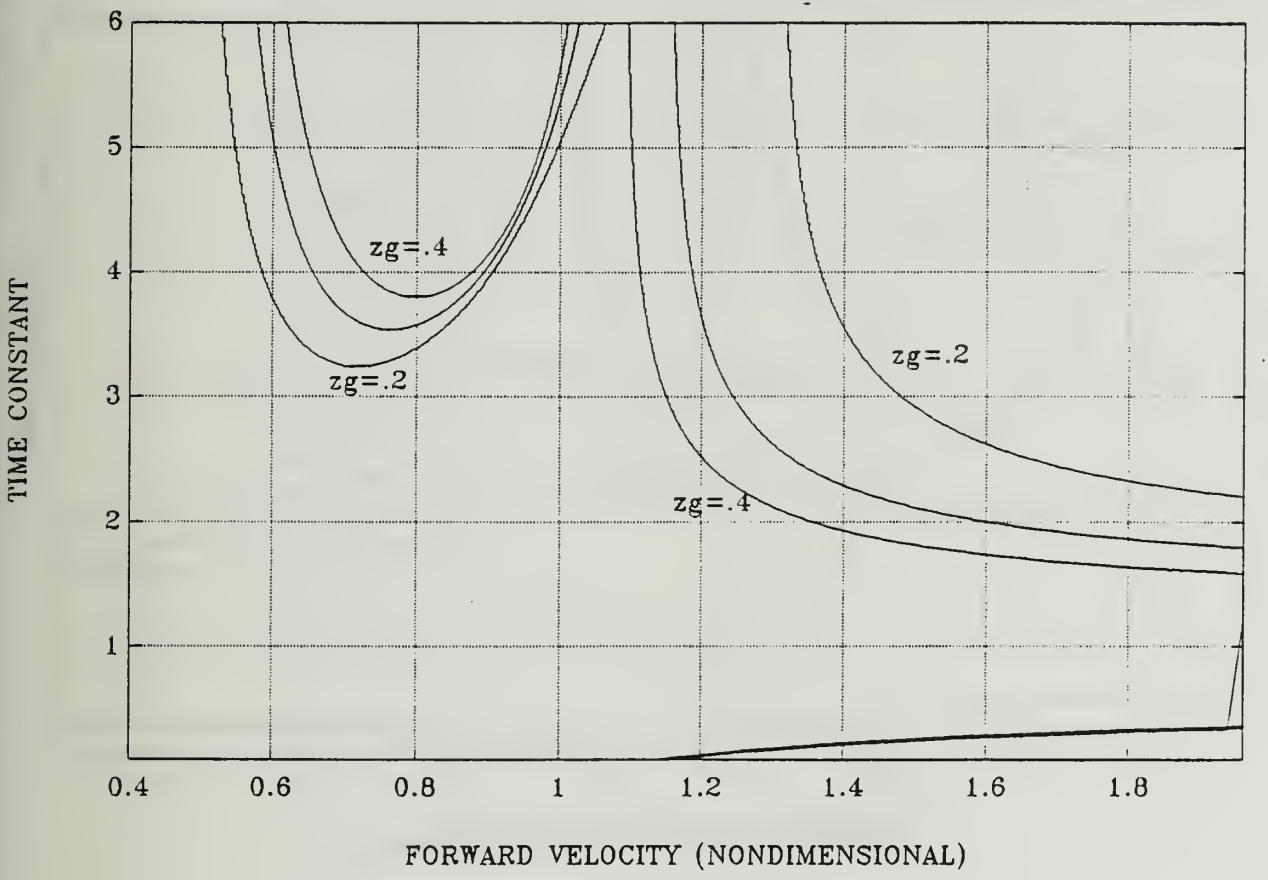

FIGURE 4.9 The effects of changing zg on the bifurcation maps, xg=.2.

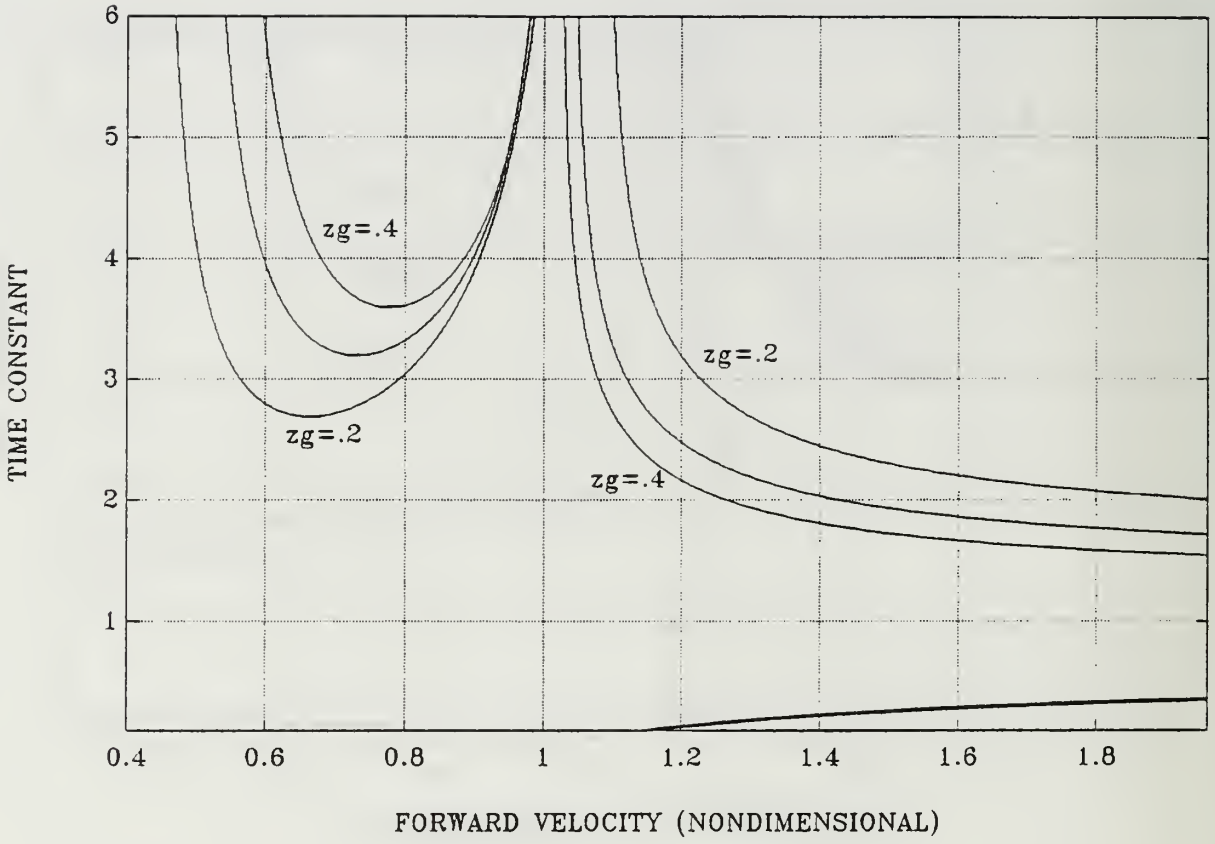

FIGURE 4.10 The effects of changing zg on the bifurcatio

maps, xg=.1.

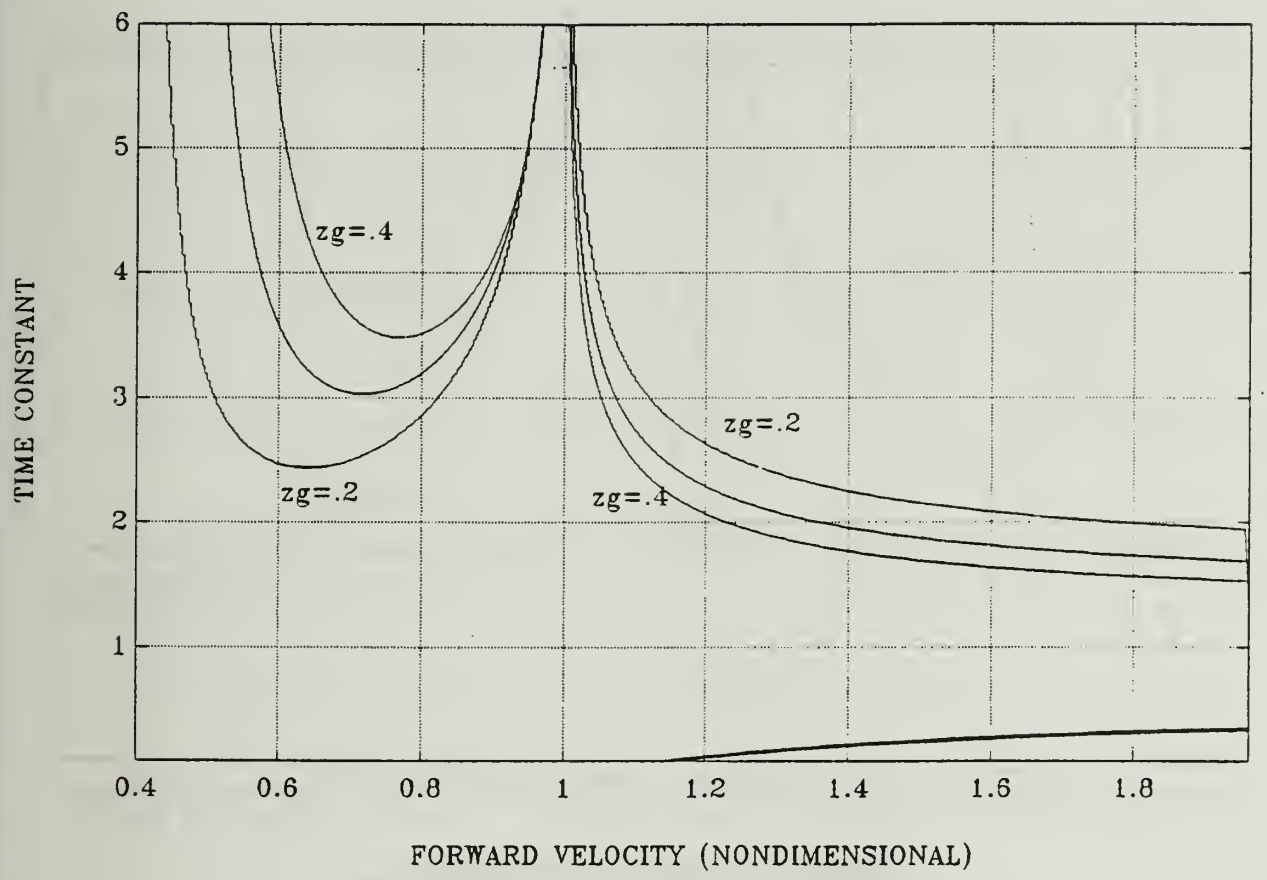

FIGURE 4.11 The effects of changing zg on the bifurcation maps, xg=0.

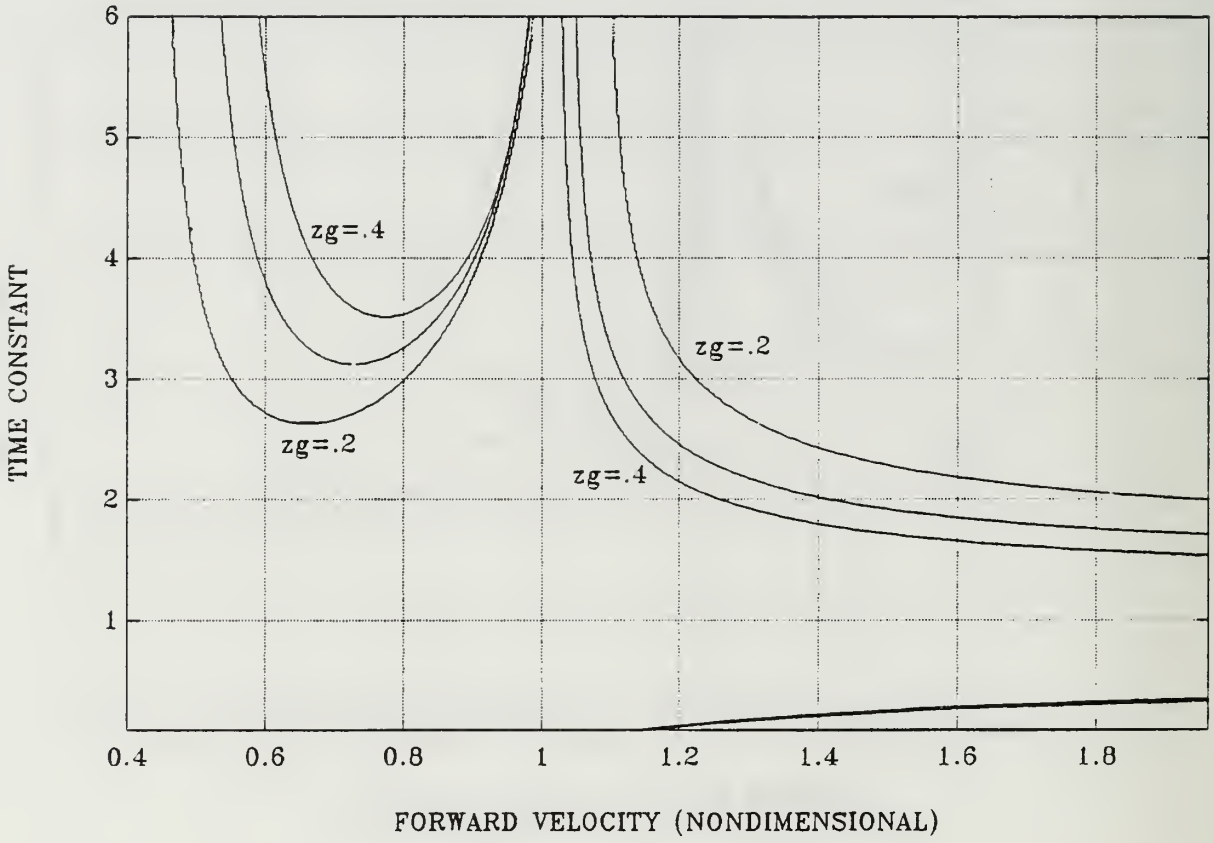

FIGURE 4.12 The effects of changing zg on the bifurcatio

maps, xg=-.1.

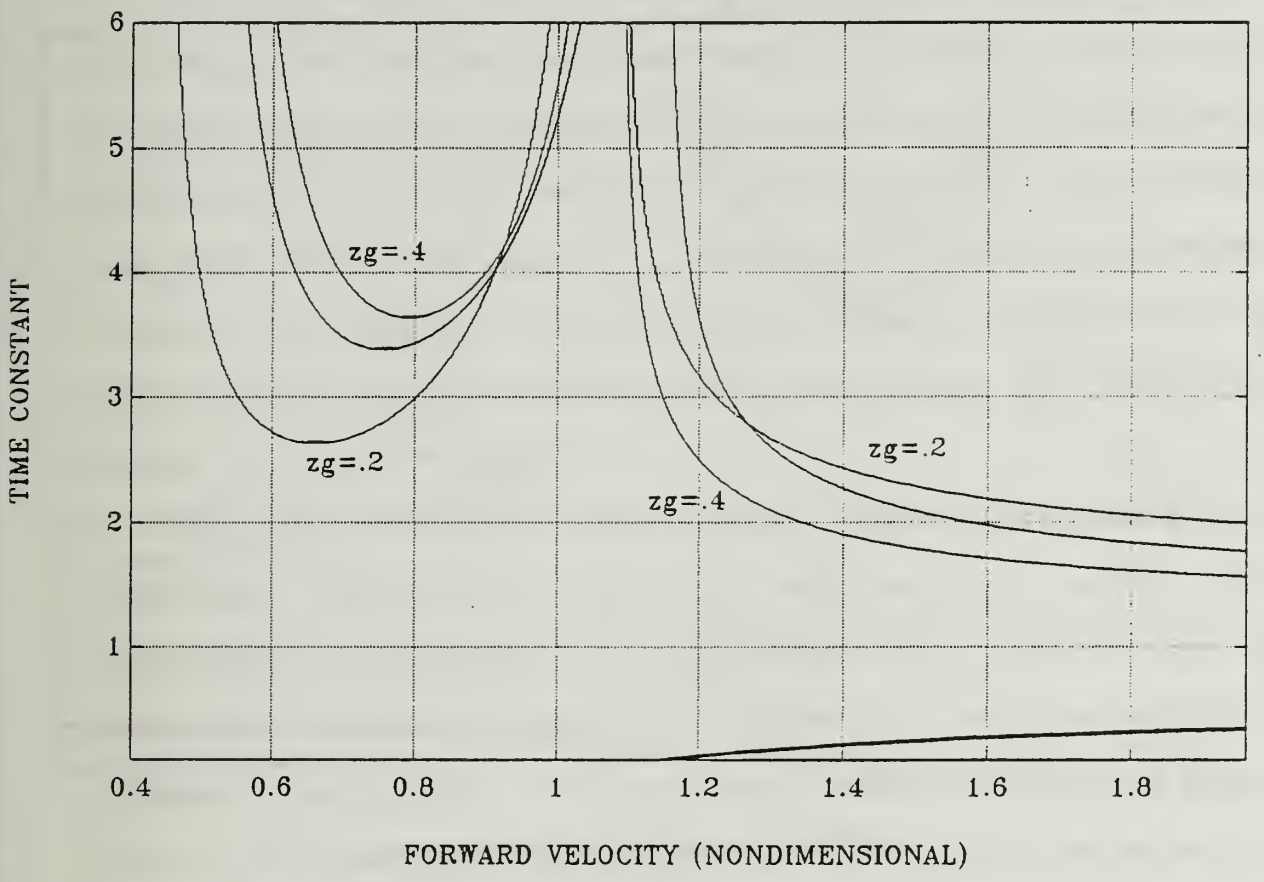

FIGURE 4.13 The effects of changing zg on the bifurcation maps, xg=-.2.

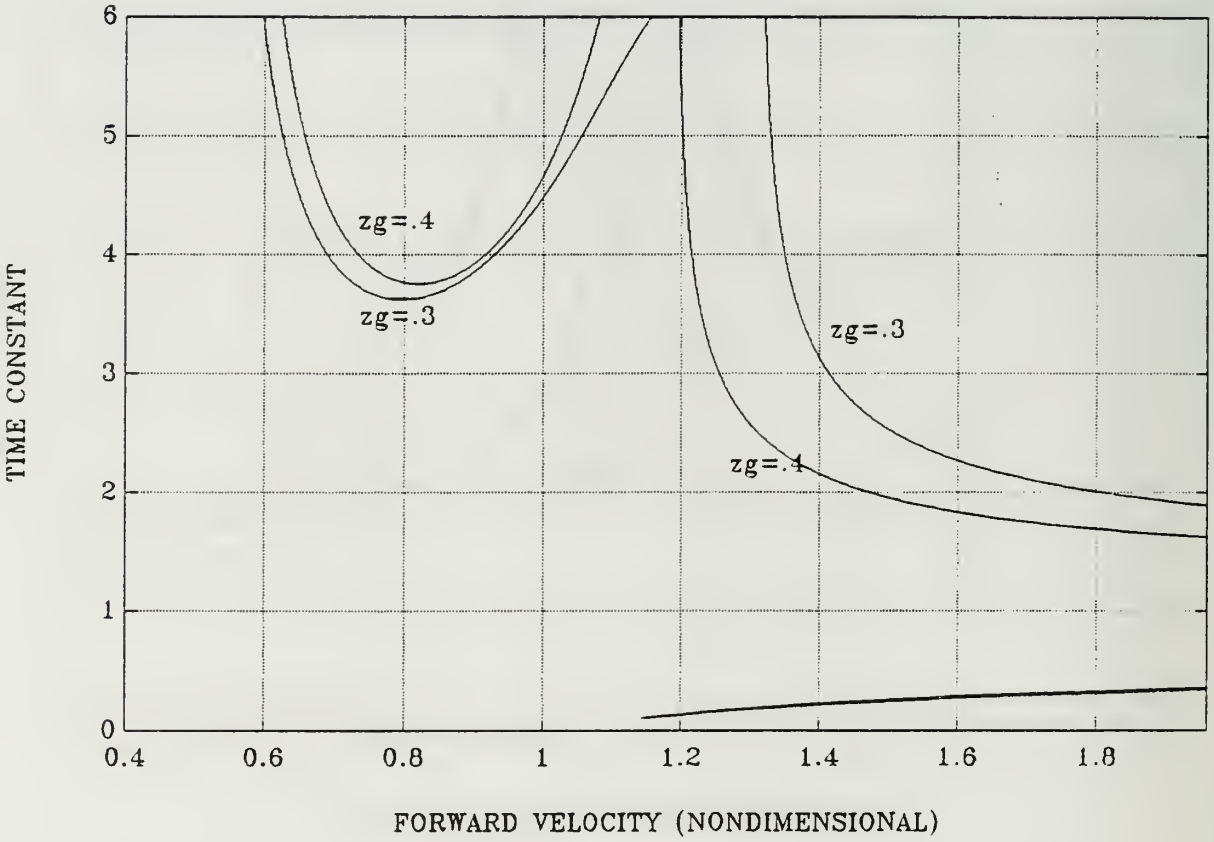

FIGURE 4.14 The effects of changing zg on the bifurcatio maps, xg=-.3.

V. CONCLUSIONS AND RECOMMENDATIONS

A. CONCLUSIONS

Hopf bifurcation analysis is a very useful design tool in the design and evaluation phase. Hopf bifurcation analysis and an identification program that can evaluate the hydrodynamic coefficients for the submersible vehicle will be very useful and save money and time by reducing the amount of model testing. An effective set of control system parameters can be generated in this process that will be optimal for the final design of the submersible.

This type of analysis can set the limits of the ranges of important parameters such as metacentric height and longitudinal separation of buoyancy/gravity centers. As we have seen changes in these two parameters can have dramatic effects on stability. It was found that the moderate speed region of stability increases with increasing metacentric height. The same is not true, however, for high speeds. The longitudinal separation of center of gravity/buoyancy can have a profound effect on stability. It was found that the vehicle may be unstable even at nominal speed. This was attributed to the fact that at high trim angles, the feedback gains which are computed at zero trim, can no longer guarantee stability.

B. RECOMMENDATIONS

The bifurcation analysis program should be expanded to evaluate the performance of the submarine including effects of external forces such as wave effects, currents, and free surface effects.

APPENDIX - BIFURCATION ANALYSIS PROGRAM

```
C    PROGRAM BIFUR1.FOR
C    BIFURCATION ANALYSIS
C    PARAMETERS ARE: TC VS. U

     IMPLICIT DOUBLE PRECISION (A-H,O-Z)
     DOUBLE PRECISION K1,K2,K3,K4,L,MQDOT,MWDOT,MQ,MW,MDS,MDB,MD,
    &              MASS,IY,P1,P2,P3,P4,XGB,ZGB
     DIMENSION A(4,4),FV1(4),IV1(4),ZZZ(4,4),WR(4),WI(4),XL(25),
    &         BR(25),VEC0(25),VEC1(25),VEC2(25)
     COMMON P1,P2,P3,P4
C
     OPEN (11,FILE='BIF1.RES',STATUS='NEW')
     OPEN (12,FILE='BIF2.RES',STATUS='NEW')
     OPEN (13,FILE='BIF3.RES',STATUS='NEW')
C  NUMERIC INFO OF DARPA SUBOFF MODEL
     WEIGHT=1556.2363
     BUO   =1556.2363
     L     =13.9792
     IY    =561.32
     G     =32.2
     MASS  =WEIGHT/G
     RHO   =1.94
     XB    =0.0
     ZB    =0.0
     CD    =0.4
```

```
      CD   =0.5*CD*RHO
C
      WRITE (*,*) 'ENTER MIN, MAX, AND INCREMENTS IN Tc (nondim)'
      READ  (*,*) TCMIN,TCMAX,ITC
      WRITE (*,*) 'ENTER MIN, MAX, AND INCREMENTS IN U (nondim)'
      READ  (*,*) UMIN,UMAX,IU
C     WRITE (*,*) 'ENTER NOMINAL SPEED'
C     READ  (*,*) U0
      WRITE (*,*) 'ENTER XG AND ZG'
      READ  (*,*) XG,ZG
      U0=9
C
      ZGB=ZG-ZB
      XGB=XG-XB
      TCMIN=TCMIN*L/U0
      TCMAX=TCMAX*L/U0
      UMIN =UMIN*U0
      UMAX =UMAX*U0
C  HYDRODYNAMIC COEFFICIENTS
      ZQDOT=-6.3300E-04*0.5*RHO*L**4
      ZWDOT=-1.4529E-02*0.5*RHO*L**3
      ZQ   = 7.5450E-03*0.5*RHO*L**3
      ZW   =-1.3910E-02*0.5*RHO*L**2
      ZDS  =-5.6030E-03*0.5*RHO*L**2
      ZDB  =-5.6030E-03*0.5*RHO*L**2
      MQDOT=-8.8000E-04*0.5*RHO*L**5
      MWDOT=-5.6100E-04*0.5*RHO*L**4
```

MQ =-3.7020E-03*0.5*RHO*L**4

MW = 1.0324E-02*0.5*RHO*L**3

MDS =-2.4090E-03*0.5*RHO*L**3

MDB = 2.4090E-03*0.5*RHO*L**3

XL(1)=0.0

XL(2)=0.1

XL(3)=0.2

XL(4)=0.3

XL(5)=0.4

XL(6)=0.5

XL(7)=0.6

XL(8)=0.7

XL(9)=1.0

XL(10)=2.0

XL(11)=3.0

XL(12)=4.0

XL(13)=7.7143

XL(14)=10.0

XL(15)=15.1429

XL(16)=16.0

XL(17)=17.0

XL(18)=18.0

XL(19)=19.0

XL(20)=20.0

XL(21)=20.1

```
      XL(22)=20.2
      XL(23)=20.3
      XL(24)=20.4
      XL(25)=20.4167
      DO 102 N=1,25
         XL(N) = (L/20.)*XL(N) - L/2.
  102 CONTINUE
      BR(1)=0.0
      BR(2)=0.485
      BR(3)=0.658
      BR(4)=0.778
      BR(5)=0.871
      BR(6)=0.945
      BR(7)=1.010
      BR(8)=1.060
      BR(9)=1.18
      BR(10)=1.41
      BR(11)=1.57
      BR(12)=1.66
      BR(13)=1.67
      BR(14)=1.67
      BR(15)=1.67
      BR(16)=1.63
      BR(17)=1.37
      BR(18)=0.919
      BR(19)=0.448
      BR(20)=0.195
```

```
      BR(21)=0.188
      BR(22)=0.168
      BR(23)=0.132
      BR(24)=0.053
      BR(25)=0.0
      DO 104 K=1,25
        VEC0(K)=BR(K)
        VEC1(K)=XL(K)*BR(K)
        VEC2(K)=XL(K)*XL(K)*BR(K)
  104 CONTINUE
      CALL TRAP(25,VECO,XL,EO)
      CALL TRAP(25,VEC1,XL,E1)
      CALL TRAP(25,VEC2,XL,E2)
C
      ALPHA=0.0
      ZD=ZDS+ALPHA*ZDB
      MD=MDS+ALPHA*MDB

C     CALCULATING THE SS PITCH ANGLE θo
C     WITH NEWTON RAPHSON METHOD
      P1= ZW*MD - MW*ZD
      P2= XG*BUO*ZD
      P3= ZG*BUO*ZD
      P4= CD*(MD*E0 - ZD*E1)
      WRITE(*,*) P1,P2,P3,P4
      EPSI=.00000001
      THETA0=0
```

```
      IF(XGB.GT.0) P4=-1*P4
18    DO 19 I=1,2000
      FT=FUNC(THETA0)
      DFT=DFUNC(THETA0)
      DELT=FT/DFT
      THETA0=THETA0-DELT
      IF(ABS(DELT)-EPSI) 20,20,19
19    CONTINUE
      FT=FUNC(THETA0)
20    WRITE(*,*) THETA0 , FT
C
      DV=(MASS-ZWDOT)*(IY-MQDOT)-(MASS*XG+ZQDOT)*(MASS*XG+MWDOT)
      A11DV=(IY-MQDOT)*(ZW-2*CD*EO*U*TAN(THETA0))
     &     +(MASS*XG+ZQDOT)*(MW+2*CD*E1*U*TAN(THETA0))

A12DV=(IY-MQDOT)*(MASS+ZQ+2*CD*E1*U*TAN(THETA0))+
     &       (MASS*XG+ZQDOT)
     &    *(MQ-MASS*XG-MASS*ZG*U*TAN(THETA0)-2*CD*E2*U*TAN(THETA0))
      A13DV=-(MASS*XG+ZQDOT)*WEIGHT
      B1DV =(IY-MQDOT)*ZD+(MASS*XG+ZQDOT)*MD
      A21DV=(MASS-ZWDOT)*(MW+2*CD*E1*U*TAN(THETA0))
     &     +(MASS*XG+MWDOT)*(ZW-2*CD*EO*U*TAN(THETA0))

A22DV=(MASS-ZWDOT)*(MQ-MASS*XG-MASS*ZG*U*TAN(THETA0)-2*CD*E2*U
     &   *TAN(THETA0))+(MASS*XG+MWDOT)*
     &        (MASS+ZQ+2*CD*E1*U*TAN(THETA0))
      A23DV=-(MASS-ZWDOT)*WEIGHT
```

```
      B2DV =(MASS-ZWDOT)*MD+(MASS*XG+MWDOT)*ZD
C
      A11=A11DV/DV
      A12=A12DV/DV
      A13=A13DV/DV
      A21=A21DV/DV
      A22=A22DV/DV
      A23=A23DV/DV
      B1 =B1DV /DV
      B2 =B2DV /DV
C
      EPS  =1.D-5
      ILMAX=1500
C
      DO 1 I=1,ITC
       WRITE (*,2001) I,ITC
       TC=TCMIN+(I-1)*(TCMAX-TCMIN)/(ITC-1)
       POLE=1.0/TC
       ALPHA3=4.0*POLE
       ALPHA2=6.0*POLE**2
       ALPHA1=4.0*POLE**3
       ALPHA0=    POLE**4
       K4=ALPHA0/((B1*A21-B2*A11)*U0**4+(B1*A23-B2*A13)*ZGB*U0**2)
       A2M=B1*U0**2
       A3M=B2*U0**2
       A0M=-(A11+A22)*U0-ALPHA3
       B1M=B2*U0**2
```

```
      B2M=(B2*A12-B1*A22)*U0**3
      B3M=(B1*A21-B2*A11)*U0**3
      B0M=(A11*A22-A21*A12)*U0**2-A23*ZGB-ALPHA2-B1*U0*U0*K4
      C1M=(B2*A11-B1*A21)*U0**3
      C2M=(B1*A23-B2*A13)*ZGB*U0**2
      C0M=(A13*A21-A23*A11)*ZGB*U0+ALPHA1-(B2+B1*A22-B2*A12)*K4*U0**3
      K2=C1M*B0M*A3M-B1M*C0M*A3M-C1M*B3M*A0M
      K2=K2/(C1M*B2M*A3M-B1M*C2M*A3M-C1M*B3M*A2M)
      K1=(C0M-C2M*K2)/C1M
      K3=(A0M-A2M*K2)/A3M
C
      DO 2 J=1,IU
       U=UMIN+(J-1)*(UMAX-UMIN)/(IU-1)
       A(1,1)=0.0D0
       A(1,2)=0.0D0
       A(1,3)=1.0D0
       A(1,4)=0.0D0
       A(2,1)=-A13*(XGB*SIN(THETA0)-ZGB*COS(THETA0))+B1*U*U*K1
       A(2,2)=A11*U  +B1*U*U*K2
       A(2,3)=A12*U  +B1*U*U*K3
       A(2,4)=       B1*U*U*K4
       A(3,1)=-A23*(XGB*SIN(THETA0)-ZGB*COS(THETA0))+B2*U*U*K1
       A(3,2)=  A21*U  +B2*U*U*K2
       A(3,3)=  A22*U  +B2*U*U*K3
       A(3,4)=  B2*U*U*K4
       A(4,1)=- U
       A(4,2)=  1.0D0
```

```
      A(4,3)= 0.0D0
      A(4,4)= 0.0D0
C
      CALL RG(4,4,A,WR,WI,0,ZZZ,IV1,FV1,IERR)
      CALL DSTABL(DEOS,WR,WI,FREQ)
C
      IF (J.GT.1) GO TO 10
      DEOSOO= DEOS
      UOO = U
      LL= 0
      GO TO 2
   10  DEOSNN = DEOS
      UNN = U
      PR= DEOSNN*DEOSOO
      IF (PR.GT.0.D0) GO TO 3
      LL = LL+1
      IF (LL.GT.3) STOP 1000
      IL = 0
      UO = UOO
      UN = UNN
      DEOSO = DEOSOO
      DEOSN = DEOSNN
    6  UL = UO
      UR = UN
      DEOSL = DEOSO
      DEOSR = DEOSN
C       U = (UL+UR)/2.D0
```

```
      ALPHA  =  (DEOSL-DEOSR)/(UL-UR)
      U =- (DEOSL-ALPHA*UL)/ALPHA
      A(1,1)  =  0.0D0
      A(1,2)  =  0.0D0
      A(1,3)  =  1.0D0
      A(1,4)  =  0.0D0
      A(2,1)  =  A13*(XGB*SIN(THETAO)-ZGB*COS(THETAO))+B1*U*U*K1
      A(2,2)  =  A11*U  +B1*U*U*K2
      A(2,3  ) =  A12*U  +B1*U*U*K3
      A(2,4  ) =    B1*U*U*K4
      A(3,1) = A23*(XGB*SIN(THETAO)-ZGB*COS(THETAO))+B2*U*U*K1
      A(3,2) =  A21*U  +B2*U*U*K2
      A(3,3) =  A22*U  +B2*U*U*K3
      A(3,4 ) =    B2*U*U*K4
      A(4,1) =- U
      A(4,2) = 1.0D0
      A(4,3) = 0.0D0
      A(4,4 )= 0.0D0
C
      CALL RG(4,4,A,WR,WI,0,ZZZ,IV1,FV1,IERR)
      CALL DSTABL(DEOS,WR,WI,FREQ)
C
      DEOSM =  DEOS
      UM = U
      PRL = DEOSL*DEOSM
      PRR =  DEOSR*DEOSM
      IF (PRL.GT.0.D0) GO TO 5
```

```
      UO = UL
      UN = UM
      DEOSO = DEOSL
      DEOSN = DEOSM
      IL = IL+1
      IF (IL.GT.ILMAX) STOP 3100
      DIF = DABS(UL-UM)
      IF (DIF.GT.EPS) GO TO 6
      U = UM
      GO TO 4
    5 IF (PRR.GT.0.D0) STOP 3200
      UO = UM
      UN = UR
      DEOSO = DEOSM
      DEOSN = DEOSR
      IL = IL+1
      IF (IL.GT.ILMAX) STOP 3100
      DIF = DABS(UM-UR)
      IF (DIF.GT.EPS) GO TO 6
      U = UM
    4 LLL = 10+LL
      WRITE (LLL,*) U/U0,TC*U0/L
    3 UOO = UNN
      DEOSOO = DEOSNN
    2 CONTINUE
    1 CONTINUE
C
```

```
2001 FORMAT (2I5)
    END


    SUBROUTINE DSTABL(DEOS,WR,WI,OMEGA)
    IMPLICIT DOUBLE PRECISION (A-H,O-Z)
    DIMENSION WR(4),WI(4)
    DEOS=-1.0D+20
    DO 1 I = 1,4
     IF (WR(I).LT.DEOS) GO TO 1
     DEOS = WR(I)
     IJ = I
   1 CONTINUE
    OMEGA = WI(IJ)
    OMEGA =  DABS(OMEGA)
    RETURN
    END
C
    SUBROUTINE TRAP(N,A,B,OUT)
C    NUMERICAL INTEGRATION ROUTINE USING THE TRAPEZOIDAL RULE
    IMPLICIT DOUBLE PRECISION (A-H,O-Z)
    DIMENSION A(1),B(1)
    N1 = N-1
    OUT = 0.0
    DO 1 I = 1,N1
      OUT1 = 0.5*(A(I)+A(I+1))*(B(I+1)-B(I))
      OUT = OUT+OUT1
```

```
 1  CONTINUE
    RETURN
    END

C   FUNCTIONS USED IN NEWTON RAPHSON ROUTINE
    FUNCTION FUNC(THETA0)
    IMPLICIT DOUBLE PRECISION (A-H,O-Z)
    COMMON P1,P2,P3,P4
    FUNC =  P1*DTAN(THETA0)  +  P2*DCOS(THETA0)  +  P3*DSIN(THETA0)
   &     + P4*(DTAN(THETA0))**2
    RETURN
    END

    FUNCTION DFUNC(THETA0)
    IMPLICIT DOUBLE PRECISION (A-H,O-Z)
    COMMON P1,P2,P3,P4
    DFUNC= P1*(1.D0/DCOS(THETA0))**2  -  P2*DSIN(THETA0) +
&   P3*DCOS(THETA0) +  P4*2.D0*DTAN(THETA0)*(1.D0/DCOS(THETA0))**2.D0
    RETURN
    END
```

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